UCRL- 93456
PREPRINT

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This paper was prepared for submittal to
the 7th International Free Electron Laser
Conference, Tahoe City, California
September 8-13, 1985

October 15, 1985

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October 15, 1985

ABSTRACT

The performance of a free electron laser strongly depends on the electron beam quality or brightness. The electron beam is transported into the free electron laser after it has been accelerated to the desired energy. Typically the maximum beam brightness produced by an accelerator is constrained by the beam brightness delivered by the accelerator injector. Thus it is important to design the accelerator injector to yield the required electron beam brightness. The DPC (Darwin Particle Code) computer code has been written to numerically model accelerator injectors. DPC solves for the transport of a beam from emission through acceleration up to the full energy of the injector. The relativistic force equation is solved to determine particle orbits. Field equations are solved for self consistent electric and magnetic fields in the Darwin approximation. DPC has been used to investigate the beam quality consequences of A-K gap, accelerating stress, electrode configuration and axial magnetic field profile.

*Performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract No. W-7405-ENG-48 and for the Department of Defense under DARPA, ARPA Order No. 4395 Amendment 31, monitored by Naval Surface Weapons Center under document number N60921-85-POW0001; SDIO/BMD-ATC MIPR #W3-RPD-53-A127; and SDIO/NSWC document number N60921-85-POW0002.

INTRODUCTION

The operation of free electron lasers place constraints on acceptable input electron beam emittance and quality. To study the effect of various accelerator injector designs on these parameters an effort is underway to numerically model the injector. There are many computer codes which have previously been used to study accelerator injectors and beam transport. The injector codes can be broadly separated into time-independent and time-dependent categories. The time-independent or steady-state codes generally fix electrode voltages and then follow macro-particles or trace single particle rays until a convergence criteria is satisfied. The object is to obtain a state which corresponds to the solution a long time after the beginning of a beam pulse. In some cases a steady-state solution is determined from a prescribed field or current distribution. The numerical model is then augmented by an analytic theory or perhaps known experimental observations. In other cases the solution is made self-consistent with fields which are applied and fields due to all current or charge sources. Use has been made of time independent results to interpret experiments and conduct scaling studies.

When physics issues arise which involve inductive effects, in particular waves, fluctuations, beam interactions with a cavity, or electromagnetic stability, it is then necessary to resort to a time-dependent simulation. The most elaborate time-dependent codes self consistently solve Maxwell's equations and the force equation for a large number of macro-particles. These particle-in-cell (PIC) simulations have been used advantageously to study a broad range of electromagnetic phenomenon. Although, the greatest amount of physics is included in these codes there are several drawbacks. In an explicit solution of Maxwell's equations the time step is restricted by a Courant condition. In practice the time step must not exceed the mesh size times the speed of light, which might typically be 10 picoseconds. This means running a PIC code can be quite expensive. A second disadvantage is the effect known as excessive bremsstrahlung. Since a PIC simulation always represents a large number of real particles by a single computational macro-particle, the numerical fluctuations are anomalously large. Consequently, unless special precautions are taken, an abnormally large amount of particle energy is radiated into electromagnetic modes.

In this work the problems of a full electromagnetic simulation are avoided by using the Darwin field approximation [1-2]. This model has been implemented for axisymmetric geometry in the DPC (Darwin Particle Code) computer code. The Darwin model is the magnetoinductive limit of Maxwell's equations, which retains the first order relativistic correction to the particle Lagrangian. This means high frequency phenomena or effects due to rapid current changes can not be studied with the Darwin model. However, because wave motion is not followed, the Courant condition of a full electromagnetic simulation can be violated. In addition, inductive effects are modeled without creating non-physical radiation. The DPC code is thus a useful implementation of a physics model which includes inductive effects missing from steady-state calculations.

The DPC code solves for beam dynamics over a distance of typically 50 cm. This includes the field emission from a cathode and acceleration up to the energy of the injector. Particle trajectories are followed from the emitting surface and past all electrodes including the anode. The DPC calculation reveals the immediate effect of parameter choices such as the A-K gap accelerating stress, electrode configuration and axial magnetic field profile. It is these considerations which must be understood to produce electron beams of high quality or brightness. DPC has the capability of external magnetic field coils, finite electrode voltage rise times, and "stair case" shaping of electrodes for geometric effects. For a particular injector design goal these capabilities permit the evaluation of the effect on brightness of non-linear applied and self fields.

DPC MODEL

DPC solves the relativistic force equation in cartesian x, y, z coordinates,

$$m \frac{d\vec{u}}{dt} = \frac{q}{c} \vec{E} + \frac{q}{c\gamma} \vec{u} \times \vec{B} \quad , \tag{1}$$

where m is particle mass, $\gamma = (1 - (v/c)^2)^{-1/2}$, v is velocity, q is charge, c is the speed of light, $\vec{u} = \gamma\vec{v}/c$, $\vec{E}$ is the electric field and $\vec{B}$ is the magnetic field. Axisymmetry is assumed so fields are only functions of r and z. Consistent with this assumption the current and charge density are obtained from the particles by spreading these quantities in theta.

Fields are obtained from Maxwell's equations in the Darwin approximation. The practical consequence of the Darwin approximation is the neglect of the solenoidal part of the displacement current. Denoting solenoidal by subscript t and irrotational by subscript ℓ Maxwell's equations in the Darwin approximation are below.

$$\nabla \cdot \vec{B}_t = 0 \tag{2a}$$

$$\nabla \times \vec{B}_t = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}_\ell}{\partial t} \tag{2b}$$

$$\nabla \cdot \vec{E}_\ell = 4\pi\rho \tag{2c}$$

$$\nabla \times \vec{E}_t = - \frac{1}{c} \frac{\partial \vec{B}_t}{\partial t} \quad . \tag{2d}$$

There are three important points to note concerning the vector decomposition of Eq. (2) into solenoidal and irrotational components and the neglect of $\partial E_t/\partial t$. First, a general magnetic field is strictly solenoidal so it plays the same role in the general equations as in the Darwin approximation. Hereafter, $\vec{B}_t$ will be denoted by $\vec{B}$. Second, it is possible to derive the continuity equation from Eq. (2b) by taking the divergence of each side. The continuity equation is not recovered in other models which neglect the entire displacement current. Third, a wave equation is usually derived by taking the curl of Faraday's equation $\nabla^2 E \sim \partial^2 E/\partial t^2$. The origin of the second time derivative term is the solenoidal part of the displacement current which is absent in the Darwin approximation. Consequently, in the Darwin approximation the fundamentally hyperbolic nature of Maxwell's equations becomes elliptic. This means DPC obtains fields by only solving elliptic equations. Another viewpoint is that the propagation speed of electromagnetic modes is taken to be infinite and thus the time-asymptotic state evolves during each time step.

The Darwin field approximation provides a set of field equations consistent with a Lagrangian correct to order β^2. There are two ways of understanding how this approximation impacts Maxwell's equations.

First the part of the Lagrangian, L related to fields consist of a sum of an electrostatic scalar potential ϕ and a vector potential $\vec{A}$.

$$L \sim \phi + \vec{\beta} \cdot \vec{A} \quad . \tag{3}$$

In general, there are relativistic corrections to both ϕ and $\vec{A}$. In the Coulomb gauge, $\nabla \cdot \vec{A} = 0$ and the potential ϕ is known to all orders in β. Thus, in this gauge L only has relativistic corrections from $\vec{A}$. The Coulomb gauge infinite media, open boundary solution for $\vec{A}$ scales like β since $\vec{J}$ scales like the velocity.

$$\vec{A}(r,t) = \frac{1}{c}\int \frac{\vec{J}\,(r',\, t - |r-r'|/c)}{|r - r'|}\, d^3r' \quad . \tag{4}$$

In Eq. (4) $\vec{J}$ represents the solenoidal right hand side of Eq. (2b). From Eq. (4) it can be seen the effect of relativity is contained in the evalution of J at a retarded time. This means the $\vec{A}$ required to cause the Lagrangian to be correct to order β^2 is just the unretarded function. To see what is neglected Eq. (4) can be expanded about the unretarded solution.

$$\begin{aligned} \vec{A}\,(r,t) &= \frac{1}{c}\int \frac{\vec{J}\,(r',t)}{|r - r'|}\, d^3r' \\ &- \frac{1}{c^2}\int \frac{\partial \vec{J}}{\partial t}\, d^3r + \ldots \\ &= \vec{A}_{\text{unretarded}} - \frac{1}{c^2}\int \frac{\partial \vec{J}}{\partial t}\, d^3r' \quad . \end{aligned} \tag{5}$$

The first neglected term in Eq. (5) scales like a wave number or inverse distance. This indicates the Darwin approximation is restricted by the allowed current variation.

The second means of seeing the implication of the Darwin approximation is to notice the consequence of the solenoidal and irrotational components in Eq. (2). Because the curl of any vector is solenoidal Eq. (2b) implies,

$$\vec{J}_\ell = -\frac{1}{4\pi}\frac{\partial \vec{E}_\ell}{\partial t} \tag{6}$$

and thus Eq. (2b) can be written,

$$\nabla \times \vec{B} = 4\pi c^{-1}\, \vec{J}_t \ . \tag{7}$$

The curl of Eq. (7) yields an elliptic equation rather than a wave equation so radiation is absent from the Darwin approximation. Likewise, the curl of Eq. (2d) yields an elliptic equation for E_t rather than a wave equation.

$$\nabla^2 \vec{E}_t = 4\pi c^{-2} \frac{\partial \vec{J}_t}{\partial t} \tag{8}$$

Equation (8) shows $\vec{E}_t$ has the time derivative of $\vec{J}_t$ as a source. This means the viability of neglecting $\partial\vec{E}_t/\partial t$ in Eq. (2b) depends on the size of the time variation of $\vec{J}_t$. If the time variation is small, $\partial\vec{E}_t/\partial t$ is insignificant and the Darwin approximation is good. Thus, the Darwin approximation is precise when $\vec{J}_t$ is <u>constant</u> and there is no radiation. The magnitude of $\vec{J}_t$ may be large in this case. It is then clear the degree of approximation depends on the amount of current variation.

DPC solves for fields on a rectangular r, z grid which contains an anode, a cathode and may also contain additional electrodes. Since axisymmetry is assumed it is not necessary to obtain the solenoidal part of the source terms to solve Eq. (2b). In the most general Darwin model because the left side of Eq. (2b) is solenoidal this step is necessary. In the DPC implementation the following two elliptic equations are solved for $\vec{B}$,

$$\Delta^* \psi_A = - 4\pi \; r \; J_\theta/c \tag{9a}$$

$$\Delta^* \psi_B = \frac{4\pi}{c} \; r \; \left(\frac{\partial J_z}{\partial r} - \frac{\partial J_r}{\partial z}\right) \quad , \tag{9b}$$

where $\psi_B = rB_\theta$, $\psi_A = rA_\theta$, A_θ is the theta component of the vector potential and $\Delta^* \equiv r^2 \; \nabla \cdot (r^{-2}\nabla$. Solving for ψ_B gives B_θ and the other two components are,

$$B_r = - \frac{1}{r} \frac{\partial \psi_A}{\partial z} \quad , \quad B_z = \frac{1}{r} \frac{\partial \psi_A}{\partial r} \; . \tag{10}$$

Usually, there are solenoids around the injector. A solenoid is modeled as a finite number of discrete axisymmetric current filaments. The magnetic field of a solenoid is obtained by summing contributions of each filament using an analytic formula. The total magnetic field is then the sum of the field from Eq. (10) plus the solenoid contribution.

The DPC electric field is calculated from equations obtained by letting $\vec{E}_\ell = - \nabla\phi$ in Eq. (2c) and taking the curl of Eq. (2d), which gives Eq. (8)

$$\nabla^2\phi = -4\pi\rho \quad . \tag{11}$$

The time derivative of the current is obtained from moments of the kinetic equation,

$$\frac{\partial\vec{J}}{\partial t} = \vec{D} + \frac{q}{m}\rho\vec{E} + \frac{q}{mc}\vec{J}\times\vec{B} \quad , \tag{12}$$

where D is the kinetic stress. To obtain the solenoidal part of $\partial\vec{J}/\partial t$ it is first written as a sum of an irrotational part plus a solenoidal part.

$$\frac{\partial\vec{J}}{\partial t} = \nabla\psi + \left(\frac{\partial\vec{J}}{\partial t}\right)_t \quad . \tag{13}$$

The divergence of both sides of Eq. (13) is taken and then a Poisson equation is solved for ψ. The desired quantity is then obtained by subtraction,

$$\left(\frac{\partial\vec{J}}{\partial t}\right)_t = \frac{\partial\vec{J}}{\partial t} - \nabla\psi \quad . \tag{14}$$

Having obtained sources for Eq (9) and (8) from particle positions and velocities it is then possible to calculate self consistent fields.

INJECTOR DESIGN STRATEGY

The DPC computer code has been used to evaluate accelerator injector brightness from the perspective of small and large area cathode emission.

It is known that brightness scales as the inverse square of beam emittance. Contributing factors to the emittance are non-linearities caused by external magnetic fields and the self fields of the beam. Near the axis the exact field can be written as an expansion consisting of linear terms plus non-linear terms which are small. Thus, with a small emitting area the radius is small and an attempt is made to reduce the effect of brightness degradation caused by non-linearities. The emitting surface can not be allowed to become too small or the output current is inadequate. Thus, it is necessary to raise the current per area emission to large values in this case.

In the designs with large area emission the non-linearity problem can clearly be troublesome. Consequently, an admission is made that some fraction of the total beam current is not useful for an FEL. Normally, a beam generated from a cathode expands due to space charge repulsion as it accelerates. To focus the beam external magnetic fields are applied. Several DPC calculations have shown significant brightness increases by relaxing the focusing magnetic field and allowing a portion of the beam to be lost. The conclusion from DPC results is that relaxing the magnetic field reduces available current, however, the phase space volume decreases more rapidly yielding a larger brightness.

Common to the strategy of small and large area emission is the issue of field stress and the injector acceleration gradient. Heuristically, the emittance scales like the product of energy and transverse velocity. Thus, to a large extent emittance is governed by the radial Lorentz force. Part of the radial Lorentz force is due to the beam radial electric field (de-focusing) and the beam theta magnetic field (focusing). These opposing contributions balance approximately as the inverse square of energy. To minimize the non-linear self field contribution to emittance, it is therefore advantageous to increase the energy as rapidly as possible. This means a high field stress aids high brightness. The maximum field stress is, however, limited by breakdown. Related to field stress is the general question of what acceleration gradient yields the highest brightness. A high brightness beam predominantly has particle trajectories in the longitudinal or z direction. Since it is desirable to have no rotation in the absence of a magnetic field the initial acceleration occurs in a small or vanishing magnetic field. The basic injector design consists of two regions. In the first region the magnetic field is increasing in magnitude and the beam is accelerated by an applied potential gradient. In the second region the magnetic field guides the beam and the potential gradient is reduced to zero. A fundamental property of a beam is that the radial space charge electric field is always larger than the self-magnetic pinching force. This means in the first region of the injector the accelerating potential gradient should be arranged to compensate for the intrinsic beam divergence. In the second region the magnetic field must be adjusted to overcome the beam divergence.

The transition from the acceleration region to the second region where the beam is drifting is accompanied by radial field aberrations. The source of the aberrations is the presence of the anode entrance. There are two means of dealing with this problem. First the magnetic profile and accelerating gradient can be arranged to cause the gradient required at the anode to be zero. This then invokes a compatible condition with the natural boundary condition which occurs at the anode. Second, a focusing cathode can be employed to offset the de-focusing effect of the anode aberrations.

REFERENCES

[1] C. W. Nielson, H. R. Lewis, Methods in Computational Physics, Vol. 16, p. 367 Academic Press, N.Y. (1976).

[2] D. W. Hewett, C. W. Nielson, J. Comp. Phys. 29 p. 219 (1978).